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Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 5 [Probability & Statistics 1]

Exam Series: May 2015 – May 2022

Format Type B:
Each question is followed by its answer scheme

Chapter 2

Permutations and combinations

79. 9709_m22_qp_52 Q: 5

A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

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(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

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The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

Answer:

Question	Answer	Marks	Guidance
(a)	${}^5C_1 \times {}^7C_4$	M1	${}^7C_4 \times k$, k integer $\geqslant 1$ Condone ${}^5P_1$ for M1 only
	175	A1	
		2	

Question	Answer	Marks	Guidance
(b)	2B 1G 2A ${}^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$ 2B 2G 1A ${}^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$ 2B 3G ${}^3C_2 \times {}^4C_3 = 12$ 3B 1G 1A ${}^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$ 3B 2G ${}^3C_3 \times {}^4C_2 = 6$	M1	${}^3C_x \times {}^4C_y \times {}^5C_z$, $x + y + z = 5$, x,y,z integers $\geqslant 1$ Condone use of permutations for this mark
		B1	2 appropriate identified outcomes correct, allow unsimplified
		M1	Summing *their* values for 4 or 5 correct identified scenarios only (no repeats or additional scenarios), condone identification by unsimplified expressions
	[Total =] 248	A1	Note: Only dependent upon M marks
		4	
(c)	$8! \times 3! \times {}^5P_2$	M1	$8! \times m$, m an integer $\geqslant 1$ Accept $8 \times 7!$ for $8!$
		M1	$3! \times n$, n an integer > 1
		M1	$p \times {}^5P_2$, $p \times {}^5C_2 \times 2$, $p \times 20$, p an integer > 1 If extra terms present, maximum 2/3 M marks available
	4 838 400	A1	Exact value required
		4	

80. 9709_s22_qp_51 Q: 1

(a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	5!	**M1**	$k!$ where $k = 5$, 6 or 7 Condone × 1 OE
	120	**A1**	
		2	
(b)	[Total no of ways =] $\frac{8!}{2!3!}$ [= 3360]	**M1**	$\frac{8!}{a!b!}$, $a = 1, 2$ $b = 1, 3$ $a \neq b$
	[With 3Es together =] $\frac{6!}{2!}$ [= 360]	**M1**	$\frac{6!}{c!}$, $c = 1, 2$ seen in an addition/subtraction
	[With 3Es not together] = 3360 – 360	**M1**	$\frac{8!}{d!e!} - \frac{6!}{f!}$ where $d, f = 1, 2$ & $e = 1, 3$
	3000	**A1**	
		4	

81. 9709_s22_qp_51 Q: 2

There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

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(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

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Answer:

Question	Answer	Marks	Guidance
(a)	$^{12}C_4 \times 2$	M1	$^gC_4 \times h$ $g = 12, 13, h = 1,2$
	990	A1	
	Alternative method for question 2(a)		
	[total – both on – neither on] $^{14}C_5 - (^{12}C_3 + ^{12}C_5) = [2002 - 220 - 792]$	M1	$^kC_5 - (^aC_3 + ^aC_5)$ $a = 12, 13$ and $k = 13, 14$
	990	A1	
		2	
(b)	[Mrs Lan plus] 2W 2M $^7C_2 \times {^6C_2} = 315$ 3W 1M $^7C_3 \times {^6C_1} = 210$ 4W $^7C_4 = 35$	M1	$^7C_r \times {^6C_{4-r}}$ for $r = 2, 3$ or 4
		B1	Outcome for one identifiable scenario correct, accept unevaluated
		M1	Add outcomes for 3 identifiable correct scenarios Note: if scenarios not labelled, they may be identified by seeing $^7C_r \times {^6C_s}$ $r + s = 4$ to imply r women and s men for both **B & M** marks only
	[Total =] 560	A1	
		4	

82. 9709_s22_qp_52 Q: 6

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

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(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

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(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

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(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	$\left[\frac{9!}{2!2!}=\right]$ 90 720	B1	
		1	
(b)	**Method 1** Arrangements Cs at ends – Arrangements Cs at ends and Os together		
	[Os not together =] $\frac{7!}{2!}-6!$ [= 2520 – 720]	M1	$\frac{w!}{2!}-y$, w = 6, 7 y an integer. Condone $2\times\left(\frac{w!}{2!}\right)-y$.
		M1	a – 6! or a – 720, a an integer resulting in a positive answer.
	1800	A1	
	Method 2 identified scenarios R ^ ^ ^ R		
	[Os not together =] $5!\times\frac{6\times5}{2!}$ =	M1	5! × b, b integer >1.
		M1	$c\times\left(\frac{6\times5}{2!}\text{ or }{}^6C_2\text{ or }\frac{{}^6P_2}{2!}\text{ or }15\right)$, c integer > 1.
	1800	A1	
		3	
Question	**Answer**	**Marks**	**Guidance**
(c)	CCO _ $\quad {}^5C_1 = 5$ CC _ _ $\quad {}^5C_2 = 10$ OOC _ $\quad {}^5C_1 = 5$ OO _ _ $\quad {}^5C_2 = 10$ C _ _ _ $\quad {}^5C_3 = 10$ O _ _ _ $\quad {}^5C_3 = 10$	B1	Correct outcome/value for 1 identified scenario. Accept unsimplified. WWW
		M1	Add 5 or 6 values of appropriate scenarios only, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified. Condone use of permutations.
	[Total =] 50	A1	
		3	
(d)	Both Os in group with a C $\quad {}^5C_2 = 10$ Both Os in group without a C $\quad {}^5C_2\times{}^3C_2 = 30$ One O in a C group, one not $\quad {}^5C_1\times{}^4C_2 = 30$ One O with each C $\quad ({}^5C_1\times{}^4C_1)\div2! = 10$	B1	A correct scenario calculated accurately. Accept unsimplified.
		M1	Add 3 or 4 correct scenario values, no incorrect scenarios, accept repeated scenarios. Accept unsimplified.
	[Total =] 80	A1	
	Alternative method for question 6(d)		
	CCO O^^ ^^^ = ${}^5C_2 = 10$ CC^ O^^ O^^ = ${}^5C_1\times{}^4C_2 = 30$ CC^ OO^ ^^^ = ${}^5C_1\times{}^4C_1 = 20$	B1	A correct scenario calculated accurately. Accept unsimplified.
	Total ways of making three groups $\frac{{}^9C_6\times{}^6C_3}{2\times2\times3} = 140$ 140 – (*their* 10+ *their* 30+ *their* 20)	M1	Total subtract 2 or 3 correct scenario values, no incorrect scenarios. Accept unsimplified.
	80	A1	
		3	

83. 9709_s22_qp_53 Q: 7

A group of 15 friends visit an adventure park. The group consists of four families.

- Mr and Mrs Kenny and their four children
- Mr and Mrs Lizo and their three children
- Mrs Martin and her child
- Mr and Mrs Nantes

The group travel to the park in three cars, one containing 6 people, one containing 5 people and one containing 4 people. The cars are driven by Mr Lizo, Mrs Martin and Mr Nantes respectively.

(a) In how many different ways can the remaining 12 members of the group be divided between the three cars? [3]

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The group enter the park by walking through a gate one at a time.

(b) In how many different orders can the 15 friends go through the gate if Mr Lizo goes first and each family stays together? [3]

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In the park, the group enter a competition which requires a team of 4 adults and 3 children.

(c) In how many ways can the team be chosen from the group of 15 so that the 3 children are all from different families? [2]

(d) In how many ways can the team be chosen so that at least one of Mr Kenny or Mr Lizo is included? [3]

Answer:

Question	Answer	Marks	Guidance
(a)	${}^{12}C_5 \times {}^{7}C_4$ [$\times {}^{3}C_3$]	M1	${}^{12}C_r \times q$, $r = 3, 4, 5$ q a positive integer > 1, no + or − .
		M1	${}^{12}C_s \times {}^{12-s}C_t$ [$\times {}^{12-s-t}C_u$] $s = 3, 4, 5$; $t = 3, 4, 5 \neq s$; $u = 3, 4, 5 \neq s, t$
	Alternative method for question 7(a)		
	$\frac{12!}{5! \times 3! \times 4!}$	M1	12! ÷ by a product of three factorials.
		M1	$\frac{n!}{5! \times 3! \times 4!}$
	[792 × 35 =] 27 720	A1	CAO
		3	

Question	Answer	Marks	Guidance
(b)	4! (Lizo) × 6! (Kenny) × 2! (Martin) × 2! (Nantes)	M1	Product involving at least 3 of 4!, 6!, 2!, 2!
	× 3! (orders of K, M and N)	M1	$w \times 3!$, w integer > 1.
	414 720	A1	WWW CAO
		3	
(c)	${}^{7}C_4$ (adults) × ${}^{4}C_1 \times {}^{3}C_1$	M1	${}^{7}C_4 \times b$, b integer > 1 no + or − .
	420	A1	
		2	
(d)	K not L ${}^{5}C_3 \times {}^{8}C_3 = 560$ L not K ${}^{5}C_3 \times {}^{8}C_3 = 560$ L and K ${}^{5}C_2 \times {}^{8}C_3 = 560$	M1	${}^{8}C_3$(or ${}^{8}P_3$) × c for one of the products or ${}^{5}C_3$ (or ${}^{5}P_3$)× c, positive integer >1 for first 2 products only.
		M1	Add 2 or 3 correct scenarios only values, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified.
	[Total or Difference=] 1680	A1	
	Alternative method for question 7(d)		
	Total no of ways – neither L nor K Total = ${}^{7}C_4 \times {}^{8}C_3 = 1960$ Neither K nor L = ${}^{5}C_4 \times {}^{8}C_3 = 280$	M1	${}^{8}C_3 \times c$, c a positive integer >1.
		M1	Subtracting the number of ways with neither from their total number of ways.
	[Total or Difference=] 1680	A1	

Question	Answer	Marks	Guidance
(d)	**Alternative method for question 7(d)**		
	Subtracting K and L from sum of K and L K ${}^{6}C_3 \times {}^{8}C_3 = 1120$ L ${}^{6}C_3 \times {}^{8}C_3 = 1120$ L and K ${}^{5}C_2 \times {}^{8}C_3 = 560$ 1120 + 1120 – 560 = 1680	M1	${}^{8}C_3 \times c$, c a positive integer >1.
		M1	Subtracting number of ways with both from sum of number of ways with K and number of ways with L.
	[Total or Difference=] 1680	A1	
		3	

84. 9709_m21_qp_52 Q: 6

(a) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR. [2]

(b) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR in which there is an R at the beginning and an R at the end, and the two As are not together. [4]

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contain both Rs and at least one A and at least one L. [4]

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Answer:

Question	Answer	Marks	Guidance
(a)	$\frac{11!}{2!2!2!}$	M1	11! alone as numerator. $2! \times m! \times n!$ on denominator, $m = 1, 2$, $n = 1, 2$. no additional terms, no additional operations.
	4989600	A1	Exact answer only.
		2	

Question	Answer	Marks	Guidance
(b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$ Number of ways of placing As in non-adjacent places = ${}^{8}C_{2}$ $\frac{7!}{2!} \times {}^{8}C_{2}$	B1	$\frac{7!}{2!} \times k$ seen, k an integer > 1.
		M1	$m \times n(n-1)$ or $m \times {}^{n}C_{2}$ or $m \times {}^{n}P_{2}$, $n = 7$, 8 or 9, m an integer > 1.
		M1	$\frac{7!}{p!} \times {}^{8}C_{2}$ or $\frac{7!}{p!} \times {}^{8}P_{2}$, p integer ⩾ 1, condone 2520×28.
	= 70560	A1	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end = $\frac{9!}{2!2!}$ Arrangements with R at ends and As together = $\frac{8!}{2!}$ With As not together = $\frac{9!}{2!2!} - \frac{8!}{2!}$	M1	$\frac{9!}{2!m!} - k$, 90720 > k integer > 1, $m = 1, 2$.
		B1	$s - \frac{8!}{2!}$, s an integer >1
		M1	$\frac{9!}{p} - \frac{8!}{q}$, p, q integers ⩾ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	A1	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	

Question	Answer	Marks	Guidance
(c)	**Method 1**		
	R R A L _ _ ${}^{5}C_{2}$ = 10 R R A L L _ ${}^{5}C_{1}$ = 5 R R A A L _ ${}^{5}C_{1}$ = 5 R R A A L L = 1	M1	${}^{5}C_{x}$ seen alone or ${}^{5}C_{x} \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0 < x < 5$ linked to an appropriate scenario.
		A1	${}^{5}C_{2} \times k$, $k = 1$ oe or ${}^{5}C_{1} \times m$, $m = 1,2$ oe alone. **SC** if ${}^{5}C_{x}$ not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. ${}^{2}C_{w} \times {}^{2}C_{x} \times {}^{2}C_{y} \times {}^{5}C_{z}$, $w+x+y+z=6$ identifies w Rs, × As and y Ls.
	[Total =] 21	A1	WWW, only dependent on 2nd M mark. Note: ${}^{5}C_{2} + {}^{5}C_{1} + {}^{5}C_{1} + 1 = 21$ is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = ${}^{7}C_{2}$	M1	${}^{7}C_{x}$ seen alone or ${}^{7}C_{x} \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$. Condone ${}^{7}P_{x}$ or ${}^{7}P_{x} \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$.
		M1	${}^{7}C_{2} \times k$, $2 \geqslant k \geqslant 1$oe
		A1	${}^{7}C_{2} \times k$, $k = 1$oe no other terms.
	[Total =] 21	A1	Value stated.
		4	

85. 9709_s21_qp_51 Q: 1

A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.

How many different selections of 5 marbles contain at least 4 marbles of the same colour? [4]

Answer:

Question	Answer	Marks	Guidance
	RRRRB $^8C_4 \times {}^4C_1 = 280$ BBBBR $^8C_1 \times {}^4C_4 = 8$ RRRRR $^8C_5 = 56$	M1	$^8C_x \times {}^4C_y$ with $x + y = 5$. x, y both integers, $1 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant 4$ condone $^8C_1 \times 1$
		A1	Two correct outcomes evaluated
		M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
	[Total =] 344	A1	WWW, only dependent on 2nd M mark
		4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

86. 9709_s21_qp_53 Q: 6

(a) How many different arrangements are there of the 11 letters in the word REQUIREMENT? [2]

(b) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which the two Rs are together and the three Es are together? [1]

(c) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs? [3]

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and at least one R? [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$\frac{11!}{2!3!}$	M1	11! alone on numerator – must be a fraction. $k! \times m!$ on denominator, $k = 1, 2$, $m = 1, 3$, 1 can be implied but cannot both = 1. No additional terms
	3326400	A1	Exact value only
		2	
(b)	8! = 40320	B1	Evaluate, exact value only
		1	
(c)	$\frac{9!}{3!} \times 7$	M1	$\frac{9!}{3!} \times k$ seen, k an integer > 0, no +, – or ÷
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	${}^9C_3 \times 7!\ (\times \frac{3!}{3!})$	M1	$9C3 \times k$ seen, k an integer > 0, no + or –
		M1	$7! \times k$ seen, , k an integer > 0, no + or –
	423360	A1	Exact value only but there must be evidence of $\times \frac{3!}{3!}$

Question	Answer	Marks	Guidance
(c) cont'd	**Alternative method for Question 6(c)**		
	$3 \times 7 \times \frac{8!}{2!}$	M1	$3 \times \frac{8!}{2!} \times k$ seen, k an integer > 0, no + or –
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	$7 \times \frac{2}{11} \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} \times \frac{1}{7} \times$ total no. of arrangements	M1	Product of correct five fractions × k seen, k an integer > 0, no + or –
		M1	7×'total no of arrangements' ×k seen, k an integer > 0, no + or –
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	No E between the Rs $- \frac{{}^6C_3 \times 3! \times 7!}{3!} = 100800$ 1E between the Rs $- \frac{{}^6C_2 \times 3! \times 7!}{2!} = 226800$ 2Es between the Rs $- {}^6C_1 \times 3! \times 7! = 90720$ 3Es between the Rs $- 7! = 5040$	M1	Finding the correct number of ways for no, 1 or 2 Es between the Rs, accept unsimplified.
		M1	Adding the number of ways for 3 or 4 correct scenarios
	$[\text{Total} = 7! \times (20 + 45 + 18 + 1) = 7! \times 84 =]\,423360$	A1	CAO
		3	

Question	Answer	Marks	Guidance
(d)	E E R _ _ $\quad {}^6C_2 = 15$ E E R R _ $\quad {}^6C_1 = 6$ E E E R _ $\quad {}^6C_1 = 6$ E E E R R $\quad {}^6C_0 = 1$	M1	Identifying four correct scenarios only.
		B1	Correct number of selections unsimplified for 2 or more scenario.
		M1	Adding the number of selections for 3 or 4 identified correct scenarios only, accept unsimplified. ${}^3C_x \times {}^2C_y \times {}^6C_z$, $x+y+z=5$ correctly identifies x Es and y Rs
	[Total =] 28	A1	WWW, only dependent upon 2nd M mark.
	Alternative method for Question 6(d) – Fixing EER first. No other scenarios can be present anywhere in solution.		
	E E R ^ ^ = ${}^8C_2$	M1	${}^8C_x$ seen alone or ${}^8C_x \times k$, , $k = 1$ or 2, $0<x<8$ Condone ${}^8P_x$ or ${}^8P_x \times k$, $k = 1$ or 2, $0<x<8$
		B1	${}^8C_2 \times k$, $k = 1$ or 2 OE
		M1	${}^8C_2 \times k$, $k = 1$ OE and no other terms
	[Total =] 28	A1	Value stated
		4	

87. 9709_w21_qp_52 Q: 2

A group of 6 people is to be chosen from 4 men and 11 women.

(a) In how many different ways can a group of 6 be chosen if it must contain exactly 1 man? [2]

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Two of the 11 women are sisters Jane and Kate.

(b) In how many different ways can a group of 6 be chosen if Jane and Kate cannot both be in the group? [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	${}^{11}C_5 \times {}^4C_1$	**M1**	${}^{11}C_5 \times {}^4C_1$ condone ${}^{11}P_5 \times {}^4P_1$ no +, –, × or ÷.
	1848	**A1**	CAO as exact.
		2	
(b)	**Method 1** [Identifying scenarios]		
	[Neither selected =] ${}^{13}C_6$ [= 1716] [Only Jane selected =] ${}^{13}C_5$ [= 1287] [Only Kate selected =] ${}^{13}C_5$ [= 1287]	**M1**	Either ${}^{13}C_6$ seen alone or ${}^{13}C_5$ seen alone or × 2 (condone ${}^{13}P_n$, n = 5,6).
	[Total =] 1716 + 1287 + 1287	**M1**	Three correct scenarios only added, accept unsimplified (values may be incorrect).
	4290	**A1**	
	Method 2 [Total number of selections – selections with Jane and Kate both picked]		
	${}^{15}C_6$ - ${}^{13}C_4$ [= 5005 – 715]	**M1**	${}^{15}C_6 - k$, k a positive integer < 5005, condone ${}^{15}P_6$.
		M1	$m - {}^{13}C_4$, m integer > 715, condone $n - {}^{13}P_4$, n > 17 160.
	4290	**A1**	
		3	
			SC Where the condition of **2(a)** is also applied in **2(b)**, the final answer is 1512 **SC M1 M1 A0** max. The method marks can be earned for the equivalent stages in each method. Method 1 ${}^4C_1 \times {}^9C_5 + {}^4C_1 \times {}^9C_4 \times 2$ Method 2 ${}^4C_1 \times {}^{11}C_5 - {}^4C_1 \times {}^9C_3$

88. 9709_w21_qp_52 Q: 4

(a) In how many different ways can the 9 letters of the word TELESCOPE be arranged? [2]

(b) In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the T and the C? [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$\frac{9!}{3!}$	M1	$\frac{9!}{e!}$, $e = 2, 3$
	60 480	A1	
		2	

Question	Answer	Marks	Guidance
(b)	$\frac{7!}{3!} \times 2 \times 6$	M1	$\frac{7!}{3!} \times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!} \times 2 \times q$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 8$ all integers.
		M1	$\frac{m!}{n!} \times p \times 6$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant p \leqslant 2$ all integers. (Accept 3P2 for 6) If **M0 M0 M0** awarded, **SC M1** for $t \times 12$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.
	Alternative method for question 4(b)		
	$\frac{{}^7P_2 \times 6! \times 2}{3!}$	M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!} \times {}^7P_2 \times q$ $m = 6,9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 2$ all integers.
		M1	$\frac{m!}{n!} \times {}^7P_r \times 2$ $m = 6, 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant r \leqslant 5$ all integers. If **M0 M0 M0** awarded, **SC M1** for $t \times 84$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.

Question	Answer	Marks	Guidance
(b)	**Alternative method for question 4(b)**		
	$\frac{7!}{3!} \times 4P2$	M1	$\frac{7!}{3!} \times k$ seen, k an integer > 0.
		M1	$t \times {}^4P_2$ or 12, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
		M1	$\frac{m!}{n!} \times 4P2$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$ all integers.
	10 008	A1	Exact value.
		4	

89. 9709_w21_qp_53 Q: 1

The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is only room for 20 people to attend.

In how many ways can the 20 people be chosen from the 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [2]

Answer:

Question	Answer	Marks	Guidance
	$^{23}C_{17}$	M1	$^{23}C_x$ or $^yC_{17}$ or $^zC_6$, x, y or z are integers no +, −, × or ÷.
	100947	A1	CAO
		2	

90. 9709_m20_qp_52 Q: 1

The 40 members of a club include Ranuf and Saed. All 40 members will travel to a concert. 35 members will travel in a coach and the other 5 will travel in a car. Ranuf will be in the coach and Saed will be in the car.

In how many ways can the members who will travel in the coach be chosen? [3]

Answer:

Question	Answer	Marks	Guidance
	$^{38}C_r$ or $^nC_{34}$	M1	Either expression seen OE, no other terms, condone x1
	$^{38}C_{34}$	A1	Correct unsimplified OE
	73815	A1	If M0, **SCB1** $^{38}C_{34}$ x k, k an integer
		3	

91. 9709_m20_qp_52 Q: 4

Richard has 3 blue candles, 2 red candles and 6 green candles. The candles are identical apart from their colours. He arranges the 11 candles in a line.

(a) Find the number of different arrangements of the 11 candles if there is a red candle at each end. [2]

(b) Find the number of different arrangements of the 11 candles if all the blue candles are together and the red candles are not together. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	R ^ ^ ^ ^ ^ ^ ^ ^ ^ R $\frac{9!}{3!6!}$	M1	9! Alone on numerator, 3! × *k* or 6! × *k* on denominator
	= 84	A1	
		2	
(b)	^ (B B B) ^ ^ ^ ^ ^ $\frac{7!}{6!}\times\frac{8\times7}{2}$	M1	$\frac{7!}{6!}\times k$ or $7k$ seen, *k* an integer > 0
		M1	$m\times n(n-1)$ or $m\times {}^nC_2$ or $m\times {}^nP_2$, *n*=7, 8 or 9, *m* an integer > 0
		M1	*n* = 8 used in above expression
	= 196	A1	
	Alternative for question 4(b)		
	[Arrangements, blues together – Arrangements with blues together and reds together =] $\frac{9!}{2!6!}-\frac{8!}{6!}$ = [252 – 56]	M1	9! Seen alone or as numerator with subtraction
		M1	8! Seen alone or as numerator in a second term and no other terms
		M1	All terms divided by 6! x *k*, *k* an integer
	= 196	A1	
		4	

92. 9709_s20_qp_51 Q: 2

(a) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

Answer:

Question	Answer	Marks
(a)	6!	**M1**
	720	**A1**
		2
(b)	Total number: $\frac{9!}{3!2!}(30240)$	**M1**
	Number with Ls together = $\frac{8!}{3!}(6720)$	**M1**
	Number with Ls not together = $\frac{9!}{3!2!} - \frac{8!}{3!}$ = 30 240 – 6720	**M1**
	23 520	**A1**
	Alternative method for question 2(b)	
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	
	$7! \times k$ in numerator, k integer ≥ 1	**M1**
	$8 \times 7 \times m$ in numerator or 8C2 $\times m$, m integer ≥ 1	**M1**
	3! in denominator	**M1**
	23 520	**A1**
		4

93. 9709_s20_qp_51 Q: 4

In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.

How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists? [4]

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Answer:

Question	Answer	Marks
	Scenarios: 2P 3V 2G $\quad {}^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $\quad {}^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $\quad {}^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $\quad {}^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (**M1** for ${}^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	**M1**
	Two unsimplified products correct	**B1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total: 13 720	**A1**
		4

94. 9709_s20_qp_52 Q: 6

(a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

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(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

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(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

Answer:

Question	*Answer*	Marks
(a)	$\frac{8!}{3!}$	M1
	6720	A1
		2

Question	Answer	Marks
(b)	Total number $= \frac{10!}{2!3!}$ (302400) (A)	B1
	With Es together $= \frac{9!}{3!}$ (60480) (B)	B1
	Es not together = *their* (A) – *their* (B)	M1
	241920	A1
	Alternative method for question 6(b)	
	_^_^_^_^_^_^_^_^_ $\frac{8!}{3!} \times \frac{9 \times 8}{2}$	
	$8! \times k$ in numerator, k integer ≥ 1, denominator ≥ 1	B1
	$3! \times m$ in denominator, m integer ≥ 1	B1
	Their $\frac{8!}{3!}$ Multiplied by ${}^9C_2$ (OE) only (no additional terms)	M1
	241920	A1
		4

Question	Answer	Marks
(c)	Scenarios: E M M M ${}^5C_0 = 1$ E M M _ ${}^5C_1 = 5$ E M _ _ ${}^5C_2 = 10$	M1
	Summing the number of ways for 2 or 3 correct scenarios	M1
	Total = 16	A1
		3

95. 9709_w20_qp_53 Q: 3

A committee of 6 people is to be chosen from 9 women and 5 men.

(a) Find the number of ways in which the 6 people can be chosen if there must be more women than men on the committee. [3]

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The 9 women and 5 men include a sister and brother.

(b) Find the number of ways in which the committee can be chosen if the sister and brother cannot both be on the committee. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	Scenarios: 6W 0M $^9C_6 = 84$ 5W 1M $^9C_5 \times {}^5C_1 = 126 \times 5 = 630$ 4W 2M $^9C_4 \times {}^5C_2 = 126 \times 10 = 1260$	**M1**	Correct number of ways for either 5 or 4 women, accept unsimplified
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios.
	Total = 1974	**A1**	
		3	
(b)	Total number of ways = $^{14}C_6$ (3003) Number with sister and brother = $^{12}C_4$ (495) Number required = $^{14}C_6 -$	**M1**	$^{14}C_6$ – a value
	$^{12}C_4 = 3003 - 495$	**M1**	$^{12}C_x$ or $^nC_4$ seen on its own or subtracted from *their* total, $x \leqslant 6$, $n \leqslant 13$
	2508	**A1**	
	Alternative method for question 3(b)		
	Number of ways with neither = $^{12}C_6 = 924$	**M1**	$^{12}C_6$ + a value
	Number of ways with either brother or sister (not both) $= {}^{12}C_5 \times 2$ (= 792×2) = 1584	**M1**	$^{12}C_x \times 2$ or $^nC_5 \times 2$ seen on its own or added to *their* number of ways with neither, $x \leqslant 5$, $n \leqslant 12$
	Number required = 924 + 1584 = 2508	**A1**	
		3	

96. 9709_m19_qp_62 Q: 7

Find the number of different arrangements that can be made of all 9 letters in the word CAMERAMAN in each of the following cases.

(i) There are no restrictions. [2]

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(ii) The As occupy the 1st, 5th and 9th positions. [1]

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(iii) There is exactly one letter between the Ms. [4]

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Three letters are selected from the 9 letters of the word CAMERAMAN.

(iv) Find the number of different selections if the three letters include exactly one M and exactly one A. [1]

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(v) Find the number of different selections if the three letters include at least one M. [3]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!3!}$	M1	9! alone on numerator, 2! and/or 3! on denominator
	= 30240	A1	Exact value, final answer
		2	
(ii)	A ^ ^ ^ A ^ ^ ^ A Arrangements = $\frac{6!}{2!} = 360$	B1	Final answer
		1	
(iii)	M ^ M ^ ^ ^ ^ ^ ^ $= \frac{7!}{3!} \times 7$	M1	7! in numerator, (considering letters not M)
		M1	Division by 3! only (removing repeated As)
		M1	Multiply by 7 (positions of M-M)
	= 5880	A1	Exact value, final answer
	Method 2 (choosing letter between Ms)		
	$1 \times \frac{6!}{2!} \times 7 + 4 \times \frac{6!}{3!} \times 7$	M1	6! in sum of 2 expressions $a6! + b6!$
		M1	Multiply by 7 in both expressions (positions of M-M)
	= 2520 + 3360	M1	$\frac{c}{2!} + \frac{d}{3!}$ seen (removing repeated As)
	= 5880	A1	Exact value

Question	Answer	Marks	Guidance
(iii)	**Method 3**		
	(MAM) ^ ^ ^ ^ ^ ^ = 7!/2! = 2520	M1	7! in numerator (considering 6 letters + block)
	(MA'M) ^ ^ ^ ^ ^ ^ = 7!/3! × 4 = 840 × 4 = 3360	M1	Division by 2! and 3! seen in different terms
	Total = 2520 + 3360	M1	Summing 5 correct scenarios only
	= 5880	A1	Exact value
		4	
'(iv)	M A ^ = $^4C_1 = 4$	B1	Final answer
		1	
(v)	M ^ ^ : $^4C_2$ = 6 M M ^ : $^4C_1$ = 4	M1	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M M A : = 1 M A A : = 1 (M A _ : $^4C_1$ = 4)	M1	Add 4 or 5 correct scenarios only
	Total = 16	A1	Value must be clearly stated
	Method 2		
	M M ^ = $^5C_1$ = 5	M1	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M ^ ^ = $^5C_2$ = 10	M1	Adding 2 or 3 correct scenarios only
	M A A = = 1 Total = 16	A1	Value must be clearly stated
		3	

97. 9709_s19_qp_61 Q: 8

Freddie has 6 toy cars and 3 toy buses, all different. He chooses 4 toys to take on holiday with him.

(i) In how many different ways can Freddie choose 4 toys? [1]

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(ii) How many of these choices will include both his favourite car and his favourite bus? [2]

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Freddie arranges these 9 toys in a line.

(iii) Find the number of possible arrangements if the buses are all next to each other. [3]

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(iv) Find the number of possible arrangements if there is a car at each end of the line and no buses are next to each other. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	($^9C_4$ =) 126	B1	
		1	
(ii)	$^7C_2$	B1	^7C_x or $^yC_2$ (implied by correct answer) or ^7P_x or ^7P_y, seen alone
	= 21	B1	correct answer
		2	

Question	Answer	Marks	Guidance
(iii)	_ C_1 (B_1 B_2 B_3) C_2 _ C_3 _ C_4 _ C_5 _ C_6	**B1**	3! or 6! seen alone or multiplied by k > 1 need not be an integer
	3! × 6! × 7	**B1**	3! and 6! seen multiplied by k > 1, integer, no division
	= 30240	**B1**	Exact value
	Alternative method for question 8(iii)		
	C_1 (B_1 B_2 B_3) C_2 C_3 C_4 C_5 C_6	**B1**	3! or 7! seen alone or multiplied by k > 1 need not be an integer
	3! × 7!	**B1**	3! and 7! seen multiplied by k > or = 1, no division
	= 30240	**B1**	Exact value
		3	
(iv)	C_1 _ C_2 _ C_3 _ C_4 _ C_5 _C_6	**B1**	6! or 4! X 6P2 seen alone or multiplied by k > 1, no division (arrangements of cars)
	6! × 5P3 *or* 6! × 5 × 4 × 3 or 6! x 3! x10	**B1**	Multiply by 5P3 oe i.e. putting Bs in between 4 of the Cs OR multiply by 3! x n where n = 7, 8, 9, 10 (number of options)
	= 43200	**B1**	Correct answer
		3	

98. 9709_s19_qp_62 Q: 7

(a) A group of 6 teenagers go boating. There are three boats available. One boat has room for 3 people, one has room for 2 people and one has room for 1 person. Find the number of different ways the group of 6 teenagers can be divided between the three boats. [3]

(b) Find the number of different 7-digit numbers which can be formed from the seven digits 2, 2, 3, 7, 7, 7, 8 in each of the following cases.

(i) The odd digits are together and the even digits are together. [3]

(ii) The 2s are not together. [4]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(a)	${}^6C_3 \times {}^3C_2 \times {}^1C_1$	**M1**	${}^6C_a \times {}^{6-a}C_b \times {}^{6-a-b}C_{6-a-b}$ seen oe ${}^{6-a-b}C_{6-a-b}$ can be implied by 1 or omission, condone use of permutations,
	$= 20 \times 3$	**A1**	Any correct method seen no addition/additional scenarios
	$= 60$	**A1**	Correct answer
	Alternative method for question 7(a)		
	$\frac{{}^6P_6}{{}^3P_3 \times {}^2P_2 \times {}^1P_1} = \frac{6!}{3! \times 2!}$	**M1**	${}^6P_6$ / (${}^nP_n$ x k) with $3 \geqslant n > 1$and $6 \geqslant$ k an integer $\geqslant 1$, not 6!/1
		A1	Correct method with no additional terms
	$= 60$	**A1**	Correct answer
		3	
(b)(i)	$\frac{4!}{3!} \times \frac{3!}{2!} \times 2$	**M1**	A single expression with either $4!/3! \times k$ or $3!/2! \times$ k, k a positive integer seen oe (condone 2 identical expressions being added)
		M1	Correctly multiplying *their* single expression by 2 or 2 identical expressions being added.
	$= 24$	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
(b)(ii)	Total no of arrangements = $\frac{7!}{2!3!}$ = 420 (A)	**B1**	Accept unsimplified
	No with 2s together = $\frac{6!}{3!}$ = 120 (B)	**B1**	Accept unsimplified
	With 2s not together: *their* (A) – *their* (B)	**M1**	Subtraction indicated, possibly by *their* answer, no additional terms present
	= 300 ways	**A1**	Exact value www
	Alternative method for question 7(b)(ii)		
	3 _ 7 _ 7 _ 7 _ 8 _		
	$\frac{5!}{3!} \times \frac{6 \times 5}{2}$	**B1**	k x 5! in numerator, k a positive integer
		B1	m x 3! In denominator, m a positive integer
		M1	*Their* 5!/3! multiplied by ${}^6C_2$ only (no additional terms)
	= 300 ways	**A1**	Exact value www
		4	

99. 9709_s19_qp_63 Q: 3

Mr and Mrs Keene and their 5 children all go to watch a football match, together with their friends Mr and Mrs Uzuma and their 2 children. Find the number of ways in which all 11 people can line up at the entrance in each of the following cases.

(i) Mr Keene stands at one end of the line and Mr Uzuma stands at the other end. [2]

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(ii) The 5 Keene children all stand together and the Uzuma children both stand together. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	9! × 2	**B1**	9! seen multiplied by $k \geqslant 1$, no addition
	= 725760	**B1**	Exact value
		2	
(ii)	Eg ($K_1K_2K_3K_4K_5$) A A A (U_1U_2) A	**B1**	2! or 5! seen mult by k > 1, no addition (arranging Us or Ks)
	= 5! × 2! × 6!	**B1**	6! Seen mult by k > 1, no addition (arranging AAAAKU)
	= 172800	**B1**	Exact value
		3	

100. 9709_s19_qp_63 Q: 4

(i) Find the number of ways a committee of 6 people can be chosen from 8 men and 4 women if there must be at least twice as many men as there are women on the committee. [3]

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(ii) Find the number of ways a committee of 6 people can be chosen from 8 men and 4 women if 2 particular men refuse to be on the committee together. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	M(8) W(4) 4 2 in $^8C_4 \times {}^4C_2 = 420$ ways 5 1 in $^8C_5 \times {}^4C_1 = 224$ ways 6 0 in $^8C_6 \times {}^4C_0 = 28$ ways	**B1**	One unsimplified product correct
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios
	Total 672 ways	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
(ii)	Total number of selections = $^{12}C_6 = 924$ (A)	**M1**	$^{12}C_x$ – (subtraction seen), accept unsimplified
	Selections with males together = $^{10}C_4 = 210$ (B)	**A1**	Correct unsimplified expression
	Total = (A) – (B) = 714	**A1**	Correct answer
	Alternative method for question 4(ii)		
	No males + Only male 1 + Only male 2 = $^{10}C_6 + {}^{10}C_5 + {}^{10}C_5$	**M1**	$^{10}C_x + 2 \text{ x } {}^{10}C_y$, $x \neq y$ seen, accept unsimplified
	= 210 + 252 + 252	**A1**	Correct unsimplified expression
	= 714	**A1**	Correct answer
	Alternative method for question 4(ii)		
	Pool without male 1 + Pool without male 2 – Pool without either male	**M1**	$2 \text{ x } {}^{11}C_x - {}^{10}C_x$
	= $^{11}C_6 + {}^{11}C_6 - {}^{10}C_6$ = 462 + 462 – 210	**A1**	Correct unsimplified expression
	= 714	**A1**	Correct answer
		3	

101. 9709_w19_qp_61 Q: 6

(i) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that all four Es are together. [1]

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(ii) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that the Ss are not next to each other. [4]

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Four letters are selected from the 12 letters of the word STEEPLECHASE.

(iii) Find the number of different selections if the four letters include exactly one S. [4]

Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!} = 181\,440$	B1	Exact value
		1	
(ii)	Total no of ways = $\frac{12!}{2!4!} = 9\,979\,200$ (A)	B1	Accept unevaluated
	With Ss together = $\frac{11!}{4!} = 1\,663\,200$ (B)	B1	Accept unevaluated
	With Ss not together = (B) – (A)	M1	Correct or $\frac{12!}{m} - \frac{8!}{n}$, m, n integers > 1 or *their* identified total – *their* identified Ss together
	8 316 000	A1	Exact value
	Alternative method for question 6(ii)		
	_ T _ E _ E _ P _ L _ E _ C _ H _ A _ E _	B1	$10! \times k$ in numerator k integer $\geqslant 1$
	$\frac{10!}{4!} \times \frac{11 \times 10}{2!}$	B1	$4! \times k$ in numerator k integer $\geqslant 1$
	$\frac{their\,10!}{their\,4!} \times {}^{11}C_2$ or ${}^{11}P_2$	M1	OE
	8 316 000	A1	Exact value
		4	

Question	Answer	Marks	Guidance
(iii)	S E E E : 1	M1	${}^6C_x$ seen alone or times $K > 1$
	S E E _ : ${}^6C_1 = 6$ S E _ _ : ${}^6C_2 = 15$ S _ _ _ : ${}^6C_3 = 20$	B1	${}^6C_3$ or ${}^6C_2$ or ${}^6C_1$ alone
	Add 3 or 4 correct scenarios	M1	No extras
	Total = 42	A1	
		4	

102. 9709_w19_qp_63 Q: 2

(i) How many different arrangements are there of the 9 letters in the word CORRIDORS? [2]

(ii) How many different arrangements are there of the 9 letters in the word CORRIDORS in which the first letter is D and the last letter is R or O? [3]

Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!3!} = 30240$	**B1**	9! Divided by at least one of 2! or 3!
		B1	Exact value
		2	
(ii)	D _ _ _ _ _ _ _ R: $\frac{7!}{2!2!} = 1260$ D _ _ _ _ _ _ _ O: $\frac{7!}{3!} = 840$	**B1**	7! Seen alone or as numerator in a term, can be multiplied not + or –
		B1	One term correct, unsimplified
	Total = 2100	**B1**	Final answer
		3	

103. 9709_w19_qp_63 Q: 3

A sports team of 7 people is to be chosen from 6 attackers, 5 defenders and 4 midfielders. The team must include at least 3 attackers, at least 2 defenders and at least 1 midfielder.

(i) In how many different ways can the team of 7 people be chosen? [4]

The team of 7 that is chosen travels to a match in two cars. A group of 4 travel in one car and a group of 3 travel in the other car.

(ii) In how many different ways can the team of 7 be divided into a group of 4 and a group of 3? [2]

Answer:

Question	Answer	Marks	Guidance
(i)	$3A\ 2D\ 2M$: ${}^6C_3 \times {}^5C_2 \times {}^4C_2$ (= 1200) $4A\ 2D\ 1M$: ${}^6C_4 \times {}^5C_2 \times {}^4C_1$ (= 600) $3A\ 3D\ 1M$: ${}^6C_3 \times {}^5C_3 \times {}^4C_1$ (= 800)	**M1**	${}^6C_x \times {}^5C_y \times {}^4C_z$, $x + y + z = 7$
		A1	2 correct products, allow unsimplified
		M1	Summing their totals for 3 correct scenarios only
	Total = 2600	**A1**	Correct answer **SC1** ${}^6C_3 \times {}^5C_2 \times {}^4C_1 \times {}^9C_1 = 7200$
		4	

Question	Answer	Marks	Guidance
(ii)	${}^7C_4 \times 1$	**B1**	${}^7C_3$ or ${}^7C_4$ seen anywhere
	35	**B1**	
		2	

104. 9709_m18_qp_62 Q: 2

A selection of 3 letters from the 8 letters of the word COLLIDER is made.

(i) How many different selections of 3 letters can be made if there is exactly one L? [1]

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(ii) How many different selections of 3 letters can be made if there are no restrictions? [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	1 L: ${}^6C_2 = 15$	**B1**	
		1	
(ii)	No L: ${}^6C_3 = 20$ (1 L: ${}^6C_2 = 15$)	**M1**	Either 0L or 2L correct unsimplified
	2 L: ${}^6C_1 = 6$	**M1**	Summing the 3 correct scenarios
	Total = 41	**A1**	
		3	

105. 9709_m18_qp_62 Q: 6

The digits 1, 3, 5, 6, 6, 6, 8 can be arranged to form many different 7-digit numbers.

(i) How many of the 7-digit numbers have all the even digits together and all the odd digits together? [3]

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(ii) How many of the 7-digit numbers are even? [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$3! \times \frac{4!}{3!} \times 2$	M1	3! oe seen multiplied by integer ⩾ 1, no addition
		M1	4!/3! oe seen multiplied by integer > 1, no addition
	= 48	A1	
		3	
(ii)	*EITHER:* Even = Total number of arrangements – Odd numbers	B1	7!/3! –
	$= 7!/3! - 3\times\frac{6\times5\times4\times3\times2\times1}{3!} = (7!/3! - 6!/2!)$ = 840 −360	B1	6!/2! OE
	= 480	B1	
	OR: No of arrangements ending in 8: $\frac{6!}{3!}$	B1	No. ending in 8 or no. ending in 6 correct unsimplified
	No ending in 6: 6!/2!	B1	Both correct and added unsimplified
	Total: $\frac{6!}{3!} + 6!/2 = 120 + 360 = 480$	B1	
		3	

106. 9709_s18_qp_61 Q: 7

Find the number of different ways in which all 9 letters of the word MINCEMEAT can be arranged in each of the following cases.

(i) There are no restrictions. [1]

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(ii) No vowel (A, E, I are vowels) is next to another vowel. [4]

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5 of the 9 letters of the word MINCEMEAT are selected.

(iii) Find the number of possible selections which contain exactly 1 M and exactly 1 E. [2]

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(iv) Find the number of possible selections which contain at least 1 M and at least 1 E. [3]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!2!} = 90720$	B1	Must see 90720
		1	
(ii)	**Method 1** ↑ * * * * * A	B1	5! seen multiplied (arrangement of consonants allowing repeats)
	No. arrangements of consonants × ways of inserting vowels =	B1	${}^6P_4$ oe (i.e. $6 \times 5 \times 4 \times 3$, ${}^6C_4 \times 4!$) seen mult (allowing repeats) no extra terms
	$\frac{5!}{2!}$ $\times \frac{{}^6P_4}{2!}$	B1	Dividing by at least one 2! (removing at least one set of repeats)
	Answer $\frac{{}^6P_4}{2!} \times \frac{5}{2} = 10\,800$	B1	Correct final answer
		4	
(iii)	${}^5C_3 = 10$	M1	${}^5C_x$ or ${}^5P_x$ seen alone, $x = 2$ or 3
		A1	Correct final answer not from ${}^5C_2$
		2	

Question	Answer	Marks	Guidance
(iv)	**Method 1** Considering separate groups	M1	Considering two scenarios of MME or EEM or MMEE with attempt, may be probs or perms
	MME** = ${}^5C_2$ = 10 MEE** = ${}^5C_2$ = 10 MMEE* = ${}^5C_1$ = 5	M1	Summing three appropriate scenarios from the four need ${}^5C_x$ seen in all of them
	ME*** = ${}^5C_3$ = 10 see **(iii)** Total = 35	A1	Correct final answer
	Method 2 Considering criteria are met if ME are chosen	M1	${}^7C_x$ only seen, no other terms
		M1	${}^xC_3$ only seen, no other terms
	ME *** = ${}^7C_3$ = 35	A1	Correct final answer
		3	

107. 9709_s18_qp_62 Q: 6

(a) Find the number of ways in which all 9 letters of the word AUSTRALIA can be arranged in each of the following cases.

(i) All the vowels (A, I, U are vowels) are together. [3]

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(ii) The letter T is in the central position and each end position is occupied by one of the other consonants (R, S, L). [3]

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(b) Donna has 2 necklaces, 8 rings and 4 bracelets, all different. She chooses 4 pieces of jewellery. How many possible selections can she make if she chooses at least 1 necklace and at least 1 bracelet? [4]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	(AAAIU) * * * * Arrangements of vowels/repeats × arrangements of (consonants & vowel group) =	M1	$k \times 5!$ (k is an integer, $k \geqslant 1$)
	$\frac{5! \times 5!}{3!}$	M1	$\frac{m}{3}!$ (m is an integer, $m \geqslant 1$) **Both** Ms can **only** be awarded if expression is **fully correct**
	= 2400	A1	Correct answer
		3	
(a)(ii)	E.g. R * * * T * * * L. Arrangements of consonants RL, RS, SL = ${}^3P_2 = 6$ Arrangements of remaining letters = $\frac{6!}{3!} = 120$	M1	$k \times \frac{6!}{3!}$ **or** $k \times {}^3P_2$ **or** $k \times {}^3C_2$ **or** $k \times 3!$ **or** $k \times 3 \times 2$ (k is an integer, $k \geqslant 1$), no irrelevant addition
	Total 120 × 6	M1	Correct unsimplified expression or $\frac{6!}{3!} \times {}^3C_2$
	= 720 ways	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
(b)	**Method 1** N(2) R(8) Br(4) 1 2 1 $= 2 \times {}^8C_2 \times 4 = 224$	M1	Multiply 3 combinations, ${}^2C_x \times {}^8C_y \times {}^4C_z$. Accept ${}^2C_1 = 2$ etc.
	2 1 1 $= 1 \times {}^8C_1 \times 4 = 32$ 1 1 2 $= 2 \times 8 \times {}^4C_2 = 96$	A1	3 or more options correct unsimplified
	2 0 2 $= 1 \times 1 \times {}^4C_2 = 6$ 1 0 3 $= 2 \times 1 \times 4 = 8$	M1	Summing *their* values of 4 or 5 legitimate scenarios (no extra scenarios)
	Total = 366 ways	A1	Correct answer
	Method 2 ${}^{14}C_4$ – (2N2R or 1N3R or 4R or 3R1B or 2R2B or 1R3B or 4B)	M1	'${}^{14}C_4 - k$' seen, k an integer from an expression containing ${}^8C_x$
	$1001 - (1 \times {}^8C_2 + 2 \times {}^8C_3 + {}^8C_4 + {}^8C_3 \times 4 + {}^8C_2 \times {}^4C_2 + 8 \times 4 + 1)$	A1	4 or more 'subtraction' options correct unsimplified, may be in a list
	1001 – (28 + 112 + 70 + 224 + 168 + 32 + 1)	M1	*Their* ${}^{14}C_4$ – [*their* values of 6 or more legitimate scenarios] (no extra scenarios, condone omission of final bracket)
	= 366	A1	Correct answer
		4	

108. 9709_s18_qp_63 Q: 7

Find the number of ways the 9 letters of the word SEVENTEEN can be arranged in each of the following cases.

(i) One of the letter Es is in the centre with 4 letters on either side. [2]

(ii) No E is next to another E. [3]

5 letters are chosen from the 9 letters of the word SEVENTEEN.

(iii) Find the number of possible selections which contain exactly 2 Es and exactly 2 Ns. [1]

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(iv) Find the number of possible selections which contain at least 2 Es. [4]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Answer:

Question	Answer	Marks	Guidance
(i)	****E**** Other letters arranged in $\frac{8!}{2!3!}$ = 3360 ways	**M1**	Mult by 8! or $^8P_8$ oe (arrangements ignoring repeats)
		A1	Correct final answer www
	OR $\frac{8\times7\times6\times5\times4\times4\times3\times2\times1}{4!2!}$ = 3360 ways	**M1**	Correct numerator (161 280)
		A1	Correct final answer www
	Total:	**2**	
(ii)	* * * * * ↑ Arrangements other letters × ways Es inserted $= \frac{5!}{2!} \times {}^6C_4 \left(\frac{5!}{2!}\times\frac{{}^6P_4}{4!}\right)$	**M1**	k mult by $^6C_4$ or $^6P_4$ oe (ways to insert Es ignoring repeats), k can = 1 or k mult by $\frac{5!}{2!}$
		M1	Correct unsimplified expression or $\frac{5!}{2!}\times{}^6P_4$
	= 900 ways	**A1**	Correct answer
	OR Total no of ways – no of ways with Es touching 9!/(4! × 2!) – … or 7 560 – … $\frac{6!}{2!} + {}^6P_2\times\frac{5!}{2!} + \frac{{}^6P_2}{2!}\times\frac{5!}{2!} + \frac{{}^6P_3}{2!\times\frac{5!}{2!}}$ = 360 + 1800 + 900 + 3600 = 6660	**M1**	7560 unsimplified – k
		M1	Attempting to find four ways of Es touching (4 Es, 3Es and a single, 2 lots of 2 Es, 2 Es and 2 singles)
	7 560 – 6 660 = 900	**A1**	Correct answer

Question	Answer	Marks	Guidance
(iv)	EE *** with no N: 1 way EEN** 3C2 or listing 3 ways EENN* 3 ways from **(iii)**	**M1**	Identifying the three different scenarios of EE, EEE or EEEE
		A1	Total no of ways with two Es (7 or 3 + 3 + 1)
	EEE** with no N: 3 ways EEEN* 3 ways EEENN 1 way	**A1**	Total no. of ways with 3 Es (7)
	EEEE* no N 3 ways EEEEN 1 way Total 18 ways	**A1**	Correct answer stated
	Method List containing ways with 2Es, 3Es and 4Es List containing at least 8 correct different ways List of all 18 correct ways Total 18	**M1**	At least 1 option listed for each of EE^^^, EEE^^, EEEE^
		A1	Ignore repeated options
		A1	Ignore repeated/incorrect options
		A1	Correct answer stated
	Total:	**4**	

109. 9709_w18_qp_61 Q: 1

9 people are to be divided into a group of 4, a group of 3 and a group of 2. In how many different ways can this be done? [3]

Answer:

Question	Answer	Marks	Guidance
	${}^9C_4 \times {}^5C_3 \times {}^2C_2$	B1	${}^9C_4$ or ${}^9C_3$ or ${}^9C_2$ seen *(1st group)*
	$=126 \times 10 \times 1$	B1	${}^{5 \text{ or } 7}C_3$ or ${}^{6 \text{ or } 7}C_4$ or ${}^{6 \text{ or } 5}C_2$ times an integer *(2nd group)*
	$=1260$	B1	Correct answer
		3	

110. 9709_w18_qp_61 Q: 3

In an orchestra, there are 11 violinists, 5 cellists and 4 double bass players. A small group of 6 musicians is to be selected from these 20.

(i) How many different selections of 6 musicians can be made if there must be at least 4 violinists, at least 1 cellist and no more than 1 double bass player? [4]

The small group that is selected contains 4 violinists, 1 cellist and 1 double bass player. They sit in a line to perform a concert.

(ii) How many different arrangements are there of these 6 musicians if the violinists must sit together? [3]

Answer:

Question	Answer	Marks	Guidance
(i)	Scenarios are: 4V + 1C + 1DB: $^{11}C_4 \times {}^5C_1 \times {}^4C_1$	M1	$^{11}C_a \times {}^5C_b \times {}^4C_c$, $a+b+c=6$,
	4V + 2C: $^{11}C_4 \times {}^5C_2$ 5V + 1C: $^{11}C_5 \times {}^5C_1$	B1	2 correct unsimplified options
	6600 + 3300 + 2310	M1	Add 2 or 3 correct scenarios only
	= 12210	A1	Correct answer
		4	
(ii)	4! × 3!	M1	k multiplied by 3! or 4!, k an integer $\geqslant 1$
		A1	Correct unsimplified expression
	= 144	A1	Correct answer
		3	

111. 9709_w18_qp_62 Q: 4

(i) Find the number of different ways that 5 boys and 6 girls can stand in a row if all the boys stand together and all the girls stand together. [3]

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(ii) Find the number of different ways that 5 boys and 6 girls can stand in a row if no boy stands next to another boy. [3]

Answer:

Question	Answer	Marks	Guidance
(i)	$5! \times 6! \times 2$	**B1**	$k\times5!$ or $m\times6!$ (k,m integer, $k,m \geqslant 1$), no inappropriate addition
		B1	$n \times 5! \times 6!$ (n integer, $n \geqslant 1$), no inappropriate addition
	$= 172800$	**B1**	Correct final answer, isw rounding (www scores B3) All marks based on their **final** answer
		3	

Question	Answer	Marks	Guidance
(ii)	… G … G … G … G … G … G … No. ways girls placed × No. ways boys placed in gaps =	**M1**	$k \times 6!$ or $k \times {}^7P_5$ (k is an integer, $k \geqslant 1$) no inappropriate add. (${}^7P_5 \equiv 7 \times 6 \times 5 \times 4 \times 3$ or ${}^7C_5 \times 5!$)
	$6! \times {}^7P_5$	**M1**	Correct unsimplified expression
	$= 1814400$	**A1**	Correct exact final answer (ignore subsequent rounding)
		3	

PP' $= \dfrac{2\times9}{2} = 9$

SS' $= \dfrac{4\times7}{2} = 14$

II' $= \dfrac{4\times7}{2} = 14$

MM' $= \dfrac{1\times10}{2} = 5$

Total number of ways $= \dfrac{10\times11}{2} = 55$

Number of ways of letters repeating $= 55 - (9 + 14 + 14 + 5) = 13$

P(Same) $= \dfrac{13}{55}$

B1 ${}^{11}C_2$ seen as the denominator of fraction (no extra terms) allow unsimplified

M1 1 – sum of 4 appropriate scenarios

A1 Correct final answer

112. 9709_w18_qp_63 Q: 1

A group consists of 5 men and 2 women. Find the number of different ways that the group can stand in a line if the women are not next to each other. [3]

Answer:

Question	Answer	Marks	Guidance
	Method 1		
	... M … M … M … M … M …	**M1**	k × 5! (120) or k × 6P2 (30), k is an integer ⩾ 1,
	No. ways men placed × No. ways women placed in gaps = $5! \times {}^6P_2$	**M1**	Correct unsimplified expression
	= 3600	**A1**	Correct answer
	Method 2		
	Number with women together = 6! × 2 (1440) Total number of arrangements = 7! (5040)	**M1**	6! × 2 or 7! – k seen, k is an integer ⩾ 1
	Number with women not together = 7! – 6! × 2	**M1**	Correct unsimplified expression
	= 3600	**A1**	Correct answer
		3	

113. 9709_m17_qp_62 Q: 5

(i) A plate of cakes holds 12 different cakes. Find the number of ways these cakes can be shared between Alex and James if each receives an odd number of cakes. [3]

(ii) Another plate holds 7 cup cakes, each with a different colour icing, and 4 brownies, each of a different size. Find the number of different ways these 11 cakes can be arranged in a row if no brownie is next to another brownie. [3]

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(iii) A plate of biscuits holds 4 identical chocolate biscuits, 6 identical shortbread biscuits and 2 identical gingerbread biscuits. These biscuits are all placed in a row. Find how many different arrangements are possible if the chocolate biscuits are all kept together. [3]

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Answer:

Question	Answer	Marks	Guidance
(i)	$^{12}C_1 + {}^{12}C_3 + {}^{12}C_5 + {}^{12}C_7 + {}^{12}C_9 + {}^{12}C_{11}$	M1	Summing at least 4 $^{12}C_x$ combinations with x = odd numbers
		A1	Correct unsimplified answer (can be implied by final answer)
	= 2048	A1	Correct answer
	Total:	3	
(ii)	$7! \times {}^8P_4$	B1	7! seen alone or multiplied only (cupcakes ordered)
		M1	multiplying by $^8P_4$ o.e (placing brownies)
	= 8467200	A1	correct answer
	Total:	3	
(iii)	9! / (6! × 2!)	B1	9! oe seen alone or as numerator
		M1	dividing by at least one of 6!,2! (removing repeated shortbread or gingerbread biscuits) ignore 4! if present
	= 252	A1	correct answer
	Total:	3	

114. 9709_s17_qp_61 Q: 7

(a) Eight children of different ages stand in a random order in a line. Find the number of different ways this can be done if none of the three youngest children stand next to each other. [3]

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(b) David chooses 5 chocolates from 6 different dark chocolates, 4 different white chocolates and 1 milk chocolate. He must choose at least one of each type. Find the number of different selections he can make. [4]

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(c) A password for Chelsea's computer consists of 4 characters in a particular order. The characters are chosen from the following.

- The 26 capital letters A to Z
- The 9 digits 1 to 9
- The 5 symbols # ~ * ? !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Chelsea can make. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	*EITHER:* e.g. xxxxx =5! for the other children	(B1	5! OE seen alone or mult by integer k ⩾ 1, no addition
	Put y in 6 ways, then 5 then 4 for the youngest children	B1	Mult by 6P3 OE
	Answer 5! × 6P3 = 14400	B1)	Correct answer
	OR: total – 3 tog – 2 tog = 8! – 6!3! – 6! × 2 × 5 × 3 = 14400	(B1	8! – 6! × k ⩾ 1seen
		B1	6!3! or 6! × 2 × 5 × 3 seen subtracted
		B1)	Correct answer
	Total:	**3**	
(b)	D W M 2 2 1 = 6C2 × 4C2 × 1 = 90	B1	One correct unsimplified option
	3 1 1 = 6C3 × 4 × 1 = 80	M1	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied
	1 3 1 = 6 × 4C3 × 1 = 24	M1	Summing the correct 3 unsimplified outcomes only
	Total=194 ways	A1	
	Total:	**4**	

Question	Answer	Marks	Guidance
(c)	C D S 2 1 1 = $^{26}C_2 \times 9 \times 5 \times 4!$ = 351 000	M1	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	1 2 1 = $26 \times {}^{9}C_2 \times 5 \times 4!$ = 112 320	M1	4 relevant products seen excluding 4! e.g. 26 × 9 × 8 × 5 or 26 × $^{9}P_2$ × 5 for 2nd outcome, condone 26 × 9 × 5 × 37 as being relevant
	1 1 2 = $26 \times 9 \times {}^{5}C_2 \times 4!$ = 56 160	M1	mult all terms by 4! or 4!/2!
	Total = 519 480	A1	
	Total:	**4**	

115. 9709_s17_qp_62 Q: 6

A library contains 4 identical copies of book A, 2 identical copies of book B and 5 identical copies of book C. These 11 books are arranged on a shelf in the library.

(i) Calculate the number of different arrangements if the end books are either both book A or both book B. [4]

(ii) Calculate the number of different arrangements if all the books A are next to each other and none of the books B are next to each other. [5]

Answer:

(i)	*EITHER*: Route 1 *A*********A* in 9! / 2!2!5! = 756 ways	(*M1	*Considering AA and BB options* with values
	*B*********B* in 9! / 4!5! = 126 ways	A1	Any one option correct
	756 + 126	DM1	*Summing their AA and BB outcomes* only
	Total = 882 ways	A1)	

Question	Answer	Marks	Guidance
	OR1: Route 2 *A*********A* in ${}^9C_5 \times {}^4C_2 = 756$ ways	(M1	*Considering AA and BB options* with values
	*B*********B* in ${}^9C_4 \times {}^5C_5 = 126$ ways	A1	Any one option correct
	756 + 126	DM1	*Summing their AA and BB outcomes* only
	Total = 882	A1)	
	Total:	4	

Question	Answer	Marks	Guidance
(ii)	*EITHER:* (The subtraction method) *A*s together, no restrictions 8! / 2!5! = 168	(*M1	*Considering all As together* – 8! seen alone or as numerator – condone × 4! for thinking A's not identical
	*A*s together and *B*s together 7! / 5! = 42	M1	*Considering all As together and all Bs together* – 7! seen alone or numerator
		M1	*Removing repeated Bs or Cs* – Dividing by 5! either expression or 2! 1st expression only – OE
	Total 168 – 42	DM1	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	A1)	
	OR1: *A*s together, no restrictions ${}^8C_5 \times {}^3C_1 = 168$	(*M1	${}^8C_5$ seen alone or multiplied
		M1	${}^7C_5$ seen alone or multiplied
	*A*s together and *B*s together ${}^7C_5 \times {}^2C_1 = 42$	M1	First expression x ${}^3C_1$ or second expression x ${}^2C_1$
	Total 168 – 42	DM1	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	A1)	
	OR2: (The intersperse method)	(M1	*Considering all "As together" with Cs* – Mult by 6!
	(AAAA)CCCCC then intersperse *B* and another *B*	M1	*Removing repeated Cs* – Dividing by 5!– [Mult by 6 implies M2]
		*M1	*Considering positions for Bs* – Mult by 7P2 oe –

Question	Answer	Marks	Guidance
	$\frac{6!}{5!} \times 7 \times 6 \div 2$	DM1	Dividing by 2! Oe – removing repeated *B*s (dependent upon **3rd M** being awarded)
	= 126	A1)	
	Total:	5	

116. 9709_w17_qp_61 Q: 6

(a) A village hall has seats for 40 people, consisting of 8 rows with 5 seats in each row. Mary, Ahmad, Wayne, Elsie and John are the first to arrive in the village hall and no seats are taken before they arrive.

(i) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John assuming there are no restrictions? [2]

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(ii) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John if Mary and Ahmad sit together in the front row and the other three sit together in one of the other rows? [4]

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(b) In how many ways can a team of 4 people be chosen from 10 people if 2 of the people, Ross and Lionel, refuse to be in the team together? [4]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	${}^{40}P_5$	M1	${}^{40}P_x$ or ${}^{y}P_5$ oe seen, can be mult by $k \geqslant 1$
	= 78 960 960	A1	
		2	
(a)(ii)	not front row e.g. WEJ** in 3× 3! = 18 ways	B1	3! seen mult by $k \geqslant 1$
	7 rows in 7 × 18= 126 ways	B1	mult by 7
	front row: e.g. *MA** in 4 × 2 = 8 ways	M1	attempt at front row arrangements and multiplying by the 7 other rows arrangements, need not be correct
	Total 126×8 = 1008	A1	
		4	
(b)	*EITHER:* e.g. *R** in ${}^8C_3$ ways = 56 ways *L** in ${}^8C_3$ = 56 ways	(M1	Considering either R or L only in team
	**** in ${}^8C_4$ = 70 ways	M1*	Considering neither in team
		DM1	summing 3 scenarios
	Total 182 ways	A1)	
	OR1: No restrictions ${}^{10}C_4$ = 210 ways	(M1	${}^{10}C_4$ – , Considering no restrictions with subtraction
	RL = ${}^8C_2$ = 28	M1*	Considering both in team
	210 – 28	DM1	subt
	= 182 ways	A1)	
Question	**Answer**	**Marks**	**Guidance**
(b)	*OR2:* R out in ${}^9C_4$ = 126 ways L out in ${}^9C_4$ = 126 ways	(M1	Considering either R out or L out
	Both out in ${}^8C_4$ = 70	M1*	Considering both out
		DM1	Summing 2 scenarios and subtracting 1 scenario
	126 + 126 – 70 = 182 ways.	A1)	
		4	

117. 9709_w17_qp_62 Q: 6

(a) Find the number of different 3-digit numbers greater than 300 that can be made from the digits 1, 2, 3, 4, 6, 8 if

(i) no digit can be repeated, [3]

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(ii) a digit can be repeated and the number made is even. [3]

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(b) A team of 5 is chosen from 6 boys and 4 girls. Find the number of ways the team can be chosen if

(i) there are no restrictions, [1]

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(ii) the team contains more boys than girls. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)(i)	*EITHER:* 3**, 4**, 6**, 8**	(M1	$^5P_2$ or $^5C_2 \times 2!$ or 5 × 4 OE (considering final 2 digits)
	options 4 × 5 × 4 = 80	M1	Mult by 4 or summing 4 options (considering first digit)
		A1)	Correct final answer
	OR: Total number of values: 6 × 5 × 4 = 120	(M1	Calculating total number of values (with subtraction seen)
	Number of values less than 300: 2 × 5 × 4 = 40	M1	Calculating number of unwanted values
	Number of evens = 120 – 40 = 80	A1)	Correct final answer
		3	

Question	Answer	Marks	Guidance
(a)(ii)	3**, 4**, 6**, 8** *EITHER:* options 4 × 6 × 4 (last)	(M1	6 linked to considering middle digit e.g. multiplied or in list
		M1	Multiply an integer by 4 × 4 (condone × 16) (No additional figures present for both M's to be awarded)
	= 96	A1)	
	OR: Total number of values 4 × 6 × 6 = 144	(M1	Calculating total number of values (with subtraction seen)
	Number of odd values 4 × 6 × 2 = 48	M1	Calculating number of unwanted values
	Number of evens = 144 – 48 = 96	A1)	
		3	
(b)(i)	252	B1	
		1	

Question	Answer	Marks	Guidance
(b)(ii)	B (6)G(4)		
	5 0 in $^6C_5\,(\times\,^4C_0) = 6 \times 1 = 6$ 4 1 in $^6C_4 \times {}^4C_1 = 15 \times 4 = 60$ 3 2 in $^6C_3 \times {}^4C_2 = 20 \times 6 = 120$	M1	Multiplying 2 combinations $^6C_q \times {}^4C_r$, $q + r = 5$, or $^6C_5$ seen alone
		M1	Summing 2 or 3 appropriate outcomes, involving perm/comb, no extra outcomes.
	Total = 186 ways	A1	
		3	

118. 9709_m16_qp_62 Q: 6

Hannah chooses 5 singers from 15 applicants to appear in a concert. She lists the 5 singers in the order in which they will perform.

(i) How many different lists can Hannah make? [2]

Of the 15 applicants, 10 are female and 5 are male.

(ii) Find the number of lists in which the first performer is male, the second is female, the third is male, the fourth is female and the fifth is male. [2]

Hannah's friend Ami would like the group of 5 performers to include more males than females. The order in which they perform is no longer relevant.

(iii) Find the number of different selections of 5 performers with more males than females. [3]

(iv) Two of the applicants are Mr and Mrs Blake. Find the number of different selections that include Mr and Mrs Blake and also fulfil Ami's requirement. [3]

Answer:

(i)	$^{15}P_5$ $= 360360$	**M1** **A1**	 2	oe, can be implied Not $^{15}C_5$ Correct answer
(ii)	$5 \times 10 \times 4 \times 9 \times 3$ $= 5400$	**M1** **A1**	 2	Mult 5 numbers Correct answer
(iii)	M(5) F(10) 3 2 $= {}^5C_3 \times {}^{10}C_2 = 450$ ways 4 1 $= {}^5C_4 \times {}^{10}C_1 = 50$ 5 0 $= {}^5C_5 \times {}^{10}C_0 = 1$ Total = 501 ways	 **M1** **M1** **A1**	 3	 Mult 2 combs, ${}^5C_x \times {}^{10}C_y$ Summing 2 or 3 two-factor options, $x + y = 5$ Correct answer
(iv)	(Couple) M(4) F(9) ManWife + 3 0 $= {}^4C_3 \times {}^9C_0 = 4$ ManWife + 2 1 $= {}^4C_2 \times {}^9C_1 = 54$ Total = 58	 **M1** **M1** **A1**	 3	 Mult 2 combs ${}^4C_x$ and ${}^9C_y$ Summing both options $x + y = 3$, gender correct Correct answer

119. 9709_s16_qp_61 Q: 6

(a) **(i)** Find how many numbers there are between 100 and 999 in which all three digits are different. [3]

(ii) Find how many of the numbers in part **(i)** are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

Answer:

(a) (i)	$9 \times 9 \times 8$ $= 648$ OR $900 - 28 \times 9 = 648$	**M1 M1** **A1** [3]	Logical listing attempt
(ii)	(7….in $1 \times 8 \times 4 = 32$ ways 8 …in $1 \times 8 \times 5 = 40$ 9… in $1 \times 8 \times 4 = 32$ Total 104 ways	**M1** **M1** **M1** **A1** [4]	Listing #s starting with 7 or 9 and ending odd
(b)	R(6) T(5) D(4) 2 2 3 = ${}^6C_2 \times {}^5C_2 \times {}^4C_3 = 600$ 2 3 2 = ${}^6C_2 \times {}^5C_3 \times {}^4C_2 = 900$ 3 2 2 = ${}^6C_3 \times {}^5C_2 \times {}^4C_2 = 1200$ Total = 2700	 **M1** **M1** **A1** **A1** [4]	 Mult 3 combs, ${}^6C_x \times {}^5C_y \times {}^4C_z$ Summing 2 or 3 three-factor outcomes can be perms, + instead of × 2 options correct unsimplified

120. 9709_s16_qp_62 Q: 7

(a) Find the number of different arrangements which can be made of all 10 letters of the word WALLFLOWER if

(i) there are no restrictions, [1]

(ii) there are exactly six letters between the two Ws. [4]

(b) A team of 6 people is to be chosen from 5 swimmers, 7 athletes and 4 cyclists. There must be at least 1 from each activity and there must be more athletes than cyclists. Find the number of different ways in which the team can be chosen. [4]

Answer:

(a) (i)	$\frac{10!}{2!3!} = 302400$	**B1** [1]	Exact value only, isw rounding

(ii)	e.g. *W******W*, **W******W, W******W**	M1	8! Seen mult or alone. Cannot be embedded (arrangements of other 8 letters).
	$\frac{8!}{3!}$ × 3(for the Ws)	M1	Dividing by 3! (removing repeated L's)
		M1	Mult by 3 (different W positions) may be sum of 3 terms
	= 20160	A1 [4]	
(b)	S(5) A(7) C(4) 1 3 2 : $5 \times {}^7C_3 \times {}^4C_2 = 1050$ 1 4 1 : $5 \times {}^7C_4 \times 4 = 700$ 2 3 1 : ${}^5C_2 \times {}^7C_3 \times 4 = 1400$ 3 2 1 : ${}^5C_3 \times {}^7C_2 \times 4 = 840$ (Outcomes : Options)	M1	Mult 3 combinations, ${}^5C_x$, ${}^7C_y$, ${}^4C_z$ (not 5 x 7 x 4)
		A1	2 correct options unsimplified
		M1	Summing only 3 or 4 correct outcomes involving combs or perms
	Total = 3990	A1 [4]	

121. 9709_s16_qp_63 Q: 6

Find the number of ways all 9 letters of the word EVERGREEN can be arranged if

(i) there are no restrictions, [1]

(ii) the first letter is R and the last letter is G, [2]

(iii) the Es are all together. [2]

Three letters from the 9 letters of the word EVERGREEN are selected.

(iv) Find the number of selections which contain no Es and exactly 1 R. [1]

(v) Find the number of selections which contain no Es. [3]

Answer:

Qu	Answer	Marks	Guidance
(i)	7560 ways	B1 [1]	
(ii)	RxxxxxxxG in $\frac{7!}{4!}$	B1	7! alone seen in num or 4! alone in denom Must be in a fraction. $\frac{7!\times 2}{4!\times 2}$ gets full marks
	= 210 ways	B1 [2]	
(iii)	eg EEEExxxxx in $\frac{6!}{2!}$	B1	6! or 5! × 6 seen in numerator or on own Can be 6! × k but not 6! ± k
	= 360 ways	B1 [2]	
(iv)	1 R eg RVG or RVN or RGN = 3	B1 [1]	
(v)	no Rs eg VGN or 3C3 ways = 1 2 Rs eg RRV or 3C1 ways = 3	M1	Summing at least 2 options for R
		A1	Correct outcome for no Rs or 2 Rs – evaluated
	Total = 7	A1 [3]	

122. 9709_w16_qp_61 Q: 5

(a) Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]

(b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.

(i) Find the number of different teams the manager can choose. [2]

(ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

Answer:

(a)	e.g. P*N*P*P*L $= \frac{5!}{3!} \times \frac{^6P_4}{2!}$ = 3600	**M1** **M1** **M1** **A1**	[4]	Mult by 5! in num Dividing by 3! or 2! Mult by $^6P_4$ oe
(b) (i)	$^7C_5 \times {}^5C_4 \times {}^2C_1 \times {}^2C_1$ = 420	**M1** **A1**	[2]	Mult 4 combs of which three are correct
(ii)	both in team $^6C_4 \times {}^4C_3 \times 2 \times 2 = 240$ 420 – 240 = 180 ways **OR** Bat in bowl out + bowl in bat out + both out $= {}^6C_4 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_4 \times 2 \times 2$ = 60 + 96 + 24 = 180 ways **OR** Bat in bowl out + bat out $= 60 + {}^6C_5 \times {}^5C_4 \times 2 \times 2 = 60 + 120 = 180$ ways	**M1** **M1** **A1** **M1** **A1** **A1** **M1** **A1 A1**	[3]	Evaluating both in team and subtracting from (i) 240 seen can be unsimplified ft their 420, their 240 summing 2 or 3 options not both in team 2 or 3 options correct unsimplified Correct ans from correct working As above, or bowl in bat out + bowl out

123. 9709_w16_qp_62 Q: 6

Find the number of ways all 10 letters of the word COPENHAGEN can be arranged so that

(i) the vowels (A, E, O) are together and the consonants (C, G, H, N, P) are together, [3]

(ii) the Es are not next to each other. [4]

Four letters are selected from the 10 letters of the word COPENHAGEN.

(iii) Find the number of different selections if the four letters must contain the same number of Es and Ns with at least one of each. [5]

Answer:

(i)	e.g. (OAEE)(CPNHGN) or cv $\frac{4!}{2!}\times\frac{6!}{2!}\times 2 = 8640$	M1 M1 A1	[3]	4!/2! or 6!/2! seen anywhere All multiplied by 2 oe
(ii)	First Method Total ways = 10!/2!2! = 907200 EE together in 9!/2! ways = 181440 EE not together = 907200 – 181440 = 725760 **OR** Second Method C P N H G N O A in 8!/2! ways Insert E in 9 ways Insert 2nd E in 8 ways, ÷ 2 Total = 8!/2!×9×8 ÷ 2 = 725760	B1 M1 M1 A1 B1 M1 M1 A1	[4]	Total ways together correct EE together attempt alone Considering total – EE together 8!/2! Seen Interspersing an E, x n where n=7,8,9. Condone additional factors. Mult by 9×8(÷2), $^9C_2$ or $^9P_2$ only oe
(iii)	First Method EN** in $^6C_2$ ways = 15 different ways EENN in 1 way Total 16 ways **OR** Second Method Listing with at least 8 different correct options Listing all correct options Total = 15 different ways EENN in 1 way Total 16 ways	M1 M1 A1 B1 A1 M1 M1 A1 B1 A1	[5]	^6C_x or $^yC_2$ seen alone or mult by $k > 1$, x<6, y>2 (1x1x) $^6C_2$ seen strictly alone or added to their EENN only Value stated or implied by final answer correct value stated Award 16 SRB2 if no method is present

124. 9709_w16_qp_63 Q: 1

A committee of 5 people is to be chosen from 4 men and 6 women. William is one of the 4 men and Mary is one of the 6 women. Find the number of different committees that can be chosen if William and Mary refuse to be on the committee together. [3]

Answer:

	total ways $^{10}C_5$ =252 MW together e.g. (MW)*** in $^8C_3$ ways = 56 MW not together = 252 – 56 = 196 ways **OR 1** 2 $^8C_4$+ $^8C_5$ 2 $^8C_4$ = 2x70=140; $^8C_5$ = 56 2 $^8C_4$+ $^8C_5$=196 **OR 2** 2 $^9C_5$ –$^8C_5$ 2 $^9C_5$ = 2 × 126 = 252; $^8C_5$ = 56 2 $^9C_5$ –$^8C_5$=196	M1 B1 A1 M1 B1 A1 M1 B1 A1	[3]	$^{10}C_5$ – … or 252– … 252 and 56 seen, may be unsimplified 2 $^nC_4$+ $^nC_5$ 140 and 56 seen may be unsimplified 2 $^9C_5$ – .. 252 and 56 seen, may be unsimplified

125. 9709_w16_qp_63 Q: 3

Numbers are formed using some or all of the digits 4, 5, 6, 7 with no digit being used more than once.

(i) Show that, using exactly 3 of the digits, there are 12 different odd numbers that can be formed. [3]

(ii) Find how many odd numbers altogether can be formed. [3]

Answer:

(i)	e.g. **5 in $^3P_2$ ways = 6	**M1**		Recognising ends in 5 or 7, can be implied
	**7 in $^3P_2$ = 6	**M1**		Summing ends in 5 + ends in 7 oe
	Total 12 AG	**A1**	[3]	Correct answer following legit working
	OR listing 457, 547, 467, 647, 567, 657, 475, 745 465, 645, 675, 765	**M1**		Listing at least 5 different numbers ending in 5
		M1		Listing at least 5 different numbers ending in 7
	Total 12 AG	**A1**		
(ii)	1 digit in 2 ways 2 digits in *5 or *7 = $^3P_1 \times 2 = 6$	**M1**		Consider at least 3 options with different number of digits. If no working, must be 3 or 4 from 2, 6, 12, 12
	4 digits in ***5 or ***7 = $^3P_3 \times 2 = 12$	**A1**		One option correct from 1, 2 or 4 digits
	Total ways = 32	**A1**	[3]	

126. 9709_s15_qp_61 Q: 7

(a) Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if

(i) there are no restrictions, [2]

(ii) the number made is an even number. [4]

(b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

Answer:

(a) (i)	$\frac{9!}{2!2!3!}$	B1	Dividing by 2!2!3!
	= 15120 ways	B1 **[2]**	Correct answer

(ii)	********3 in $\frac{8!}{2!2!3!}$ = 1680 ways ********7 in $\frac{8!}{2!3!}$ = 3360 ways Total even = 15120 – 1680 – 3360 = 10080 ways OR ********2 in 8!/2!3! = 3360 ways ********6 in 8!/2!2!3! = 1680 ways ********8 in 8!/2!2!2! = 5040ways Total = 10080 ways OR "15120" ×6/9 = 10080	B1 B1 M1 A1 **[4]** B1 B1 M1 A1 M2 A2	Correct ways end in 3 Correct ways end in 7 Finding odd and subt from 15120 or their **(i)** Correct answer One correct way end in even correct way end in another even Summing 2 or 3 ways Correct answer Mult their **(i)** by 2/3 oe Correct answer
(b)	T(3) S(6) G(14) 1 1 3 in $3\times6\times{}^{14}C_3 = 6552$ 1 3 1 in $3\times{}^6C_3\times14 = 840$ 3 1 1 in $1\times6\times14 = 84$ 2 2 1 in ${}^3C_2\times{}^6C_2\times14 = 630$ 2 1 2 in ${}^3C_2\times6\times{}^{14}C_2 = 1638$ 1 2 2 in $3\times{}^6C_2\times{}^{14}C_2 = 4095$ Total ways = 13839 (13800)	M1 M1 M1 B1 A1 **[5]**	Mult 3 (combinations) together assume $6 = {}^6C_1$ etc Listing at least 4 different options Summing at least 4 different options At least 3 correct numerical options Correct answer

127. 9709_s15_qp_62 Q: 6

(a) Find the number of different ways the 7 letters of the word BANANAS can be arranged

(i) if the first letter is N and the last letter is B, [3]

(ii) if all the letters A are next to each other. [3]

(b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cannot both be in the group together. [3]

Answer:

(a) (i)	N*****B Number of ways = $\frac{5!}{3!}$ = 20	B1 B1 B1 **3**	5! Seen in num oe or alone mult by $k \geqslant 1$ 3! Seen in denom can be mult by $k \geqslant 1$ Correct final answer
(ii)	B(AAA)NNS Number of ways = $\frac{5!}{2!}$ or ${}^5P_3$ = 60	M1 M1 A1 **3**	5! seen as a num can be mult by $k \geqslant 1$ Dividing by 2! Correct final answer
(b)	${}^{14}C_9$ total options = 2002 T and M both in ${}^{12}C_7$ = 792 Ans 2002 – 792 = 1210 OR Neither in ${}^{12}C_9$ = 220 One in ${}^{12}C_8$ = 495 Other in ${}^{12}C_8$ = 495	M1 B1 A1 **3** M1 B1	${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt ${}^{12}C_7$ (792) seen Correct final answer Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9$ + ${}^{12}P_8$ + ${}^{12}P_8$ here only Second correct option seen accept another 495 or if M1 not awarded, any correct option

128. 9709_s15_qp_63 Q: 7

Rachel has 3 types of ornament. She has 6 different wooden animals, 4 different sea-shells and 3 different pottery ducks.

(i) She lets her daughter Cherry choose 5 ornaments to play with. Cherry chooses at least 1 of each type of ornament. How many different selections can Cherry make? [5]

Rachel displays 10 of the 13 ornaments in a row on her window-sill. Find the number of different arrangements that are possible if

(ii) she has a duck at each end of the row and no ducks anywhere else, [3]

(iii) she has a duck at each end of the row and wooden animals and sea-shells are placed alternately in the positions in between. [3]

Answer:

(i)	W S D 1 1 3 = $6\times4\times{}^3C_3$ = 24 1 3 1 = $6\times{}^4C_3\times3$ = 72 3 1 1 = ${}^6C_3\times4\times3$ = 240 1 2 2 = $6\times{}^4C_2\times{}^3C_2$ = 108 2 1 2 = ${}^6C_2\times4\times{}^3C_2$ = 180 2 2 1 = ${}^6C_2\times{}^4C_2\times3$ = 270 Total = 894	M1 M1 M1 B1 A1 **[5]**	Listing at least 4 different options Mult 3 (combs) together assume 6 = ${}^6C_1$, $\Sigma r=5$ Summing at least 4 different evaluated/unsimplified options >1 At least 3 correct unsimplified options Correct answer
(ii)	${}^3P_2\times{}^{10}P_8$ = 10886400	B1 B1 B1 **[3]**	${}^3P_2$ oe seen multiplied either here or in (iii) $k{}^{10}P_x$ seen or $k{}^yP_8$ with no addition, $k\geqslant1$, $y>8$, $x<10$ Correct answer, nfww
(iii)	DSWSWSWSWD or DWSWSWSWSD D in ${}^3P_2$ ways = 6 S in ${}^4P_4$ ways = 24 W in ${}^6P_4$ = 360 Swap SW in 2 ways Total = 103680 ways	B1 B1 B1 **[3]**	If ${}^3P_2$ has not gained credit in (ii) may be awarded ${}^4P_4$ or ${}^6P_4$ oe seen multiplied or common in all terms (no division) Mult by 2 (condone 2!) Correct answer, 3sf or better, nfww

129. 9709_w15_qp_61 Q: 5

(a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged

(i) if all the letters E are together, [3]

(ii) if the T is at one end and there is an S at the other end. [3]

(b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

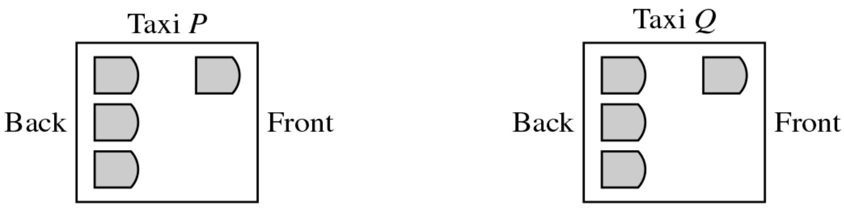
Answer:

(i)	**5 (i)** eg **(EEEE)*** Number of ways = $\frac{6!}{2!2!}$ = 180	**M1** **M1** **A1** [3]	Mult by 6! oe Dividing by 2!2! oe Correct answer
(ii)	S*******T or T*******S Number of ways = $\frac{7!}{4!2!}\times 2$ = 210	**M1** **M1** **A1** [3]	Mult by 7! Or dividing by one of 2! or 4! Mult by 2 Correct answer
(iii)	exactly one E in ${}^6C_3$ ways = 20	**M1** **M1** **A1** [3]	${}^6C_x$ as a single answer ${}^xC_3$ as a single answer correct answer

130. 9709_w15_qp_62 Q: 4

A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

Answer:

(i)	Two in same taxi: $^6C_2 \times {}^4C_4 \times 2$ or $^6C_2 + {}^6C_4$	**M1**	$^6C_4$ or $^6C_2$ oe seen anywhere
		M1	'something' ×2 only or adding 2 equal terms
	= 30	**A1** 3	Correct final answer
(ii)	MJS in taxi	**M1**	$^5P_1$, $^5C_1$ or 5 seen anywhere
	$(^5C_1 \times 2 \times 2) \times {}^4P_4$	**M1**	Mult by 2 or 4 oe
		M1	Mult by $^4P_4$ oe eg 4! or $4 \times {}^3P_3$ or can be part of 5!
	= 480	**A1** 4	Correct final answer

131. 9709_w15_qp_63 Q: 5

(a) Find the number of different ways that the 13 letters of the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other. [3]

(b) There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. [5]

Answer:

(a)	e.g. **(AAOOOI)***** $\frac{8!}{2!2!}\times\frac{6!}{2!3!}=604800$	**B1** **M1** **A1** 3	8! (8 × 7!) or 6! seen anywhere, either alone or in numerator) Dividing by at least 3 of 2!2!2!3! (may be fractions added) Correct answer
(b)	C(7) E(6) A(4) 1 1 2 $= 7 \times 6 \times {}^4C_2 = 252$ 1 2 1 $= 7 \times {}^6C_2 \times 4 = 420$ 1 3 0 $= 7 \times {}^6C_3 \times 1 = 140$ 2 1 1 $= {}^7C_2 \times 6 \times 4 = 504$ 2 2 0 $= {}^7C_2 \times {}^6C_2 \times 1 = 315$ 3 1 0 $= {}^7C_3 \times 6 \times 1 = 210$ Total = 1841	**M1** **A1** **M1*** **DM1** **A1** 5	Mult 3 appropriate combinations together assume $6={}^6C_1$, $1={}^4C_0$ etc., $\sum r=4$, C&E both present At least 3 correct unsimplified products Listing at least 4 different correct options Summing at least 4 outcomes, involving 3 combs or perms, $\sum r=4$ Correct answer SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for ${}^7C_1\times{}^6C_1\times$(sum of at least 4 outcomes)